O'Connell & Shoemaker

Shearing Strength of Concrete

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SHEARING STRENGTH OF CONCRETE

BY

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This is to certify that the following thesis prepared under the direction of Professor A. N. Talbot, Head of the Department of Municipal and Sanitary Engineering, by

CHARLES SLADE O'CONNELL and JOHN EARL SHOEMAKER

entitled SHEAR IN CONCRETE

is accepted by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering

Ira O. Baker.

Head of Department of Civil Engineering

SHEARING STRENGTH OF CONCRETE.

-o-

The determination of the shearing strength of concrete is of great importance because of the necessity of providing for the shearing forces to which this material may be subjected. But few tests for shear have been made, however, partly because of the lack of appreciation of its value, and partly because of the fact that there was no special necessity for its being considered until reinforced concrete beams became a prominent engineering material.

As early as 1879 tests upon mortar specimens were made by Prof. Bauschinger, who found that the shearing strength was about 20% in excess of the tensile. These tests were made upon flexure specimens, 2.4" x 4.8" x 1'-0", the exact method of testing not being given. Prof. Cecil B. Smith also made a series of tests by cementing three bricks together with mortar, the middle one projecting above the other two, and the load being applied in such a manner as to avoid transverse bending. However, these tests are of little value in regard to the shearing strengths of concrete, because of the dissimilarity of the two materials.

Many assumptions have been made about the relative values of the shearing strengths of concretes. M. Christophe, believing from his investigations that the tensile strength was greater that the shearing strength, allowed a working value of shear of only 21 to 35 pounds per square inch; while M. Feret, concluding from experiments that the shearing strength was 16% to 20% of the compressive, and M. Considere, that the shearing strength, as determined by his tests, was 20% to 30% greater that the tensile, agreed in allowing as a working value of shear, one-eighth of the allow-

able compressive stress, or about 50 pounds per square inch. These tests, or similar ones, probably form the basis of the requirement in the New York Regulations for Concrete, which state that the shearing strains shall not exceed 50 pounds per squate inch; also the Prussian Regulations which require that the unit shear shall not exceed 64 pounds per square inch or one-fifth of the ultimate shearing strength.

Recent tests by Prof. Charles M. Spofford at the Massachusetts Institute of Technology indicate that the shearing strength of concrete is about one-half of the compressive strength. His specimens were 5 inches in diameter by 15-1/2 inches long, and in testing were held firmly in cylindrical bearings 5-1/2 inches apart, the load being applied from above through a half cylinder bearing 5-7/16 inches in length.

These and similar experiments have been conducted in connection with numerous tests of reinforced concrete, which has so recently become an important structural material. In many flexure tests on reinforced concrete beams it has been noticed that cracks have appeared, extending diagonally downward and outward from the load points. These cracks were at first loosely called shear cracks, but a later and better explanation is that failure along these diagonal lines is by increased tension resulting from the vertical shearing and horizontal tensile stresses. It is also possible that there may have been, in some cases, a slipping of the reinforcement rods.

It is the purpose of this thesis to investigate the question of shear in concrete; to determine the ultimate shearing strength; and to compare the shearing and compressive strengths. Some comparison will be made of the relative strengths of concretes of

different ages; also a comparison of blocks stored in air with those stored in water. Some tests will also be made on reinforced concrete beams, as it is desired to determine the nature of the stresses causing the failure along the so-called shear cracks.

The general length of time between making a testing of specimens will be sixty days, a very few tests being made at an earlier date.

References used:

"Cement, Mortar Concrete", by Falk pg 27.

"Reinforced Concrete", by Marsh pg 222.

"Concrete, Plain and Reinforced", by Taylor and Thompson pg 270.

"Cement and Concrete", by Sabin ppg 90, 405.

DESCRIPTION OF MATERIAL.

As the material used in making the test pieces for this investigation are fully described in Mr. E. T. Renner's thesis, only a summary will be given here. (Thesis referred to later.)

STONE: The stone used was Kankakee limestone, the sizes were such that 96% passed a 1 inch screen and 8% was retained upon a one-fourth inch mesh. The analysis showed 49% voids.

SAND: The sand was clear and sharp and was screened through a one-fourth inch mesh before using. It weighed 10.3 pounds per cubic foot loose and contained 28% voids.

CEMENT: All cement used for these tests consisted of a mixture of fine standard brands of Portland cement in equal proportions. This material was furnished by the makers and was mixed at the mills. The tensile strength for seven day tests was 723 ± 12 pounds per square inch for neat cement briquetts and 354 ± 12 for 1:3 mortar.

CONCRETE: The concrete was mixed by hand in the proportions 1:3:6 by loose volume. A moderately wet concrete was used, the per cent. of water being about 9% of the total weight of the dry material.

STEEL: Two grades of steel were furnished, one by the Carnegie Steel Company, the other by the Crucible Steel Company of America. The former had an elastic limit of 34,000 pounds per square inch, and will be referred to as low steel. The latter had an elastic limit of 52,900, and will be referred to as high steel.

DESCRIPTION OF TEST SPECIMENS.

(a) Plain Concrete Shear Specimens.

As it is one of the purposes of this investigation to obtain the ultimate shearing strength of concrete, is was necessary for success that a form of specimen be determined upon, in which the failure would be due to shear alone, and not to a combination of shear and flexure. This difficulty has always interfered most seriously in all experiments along this line, and considerable thought was given to the design of the form of test piece.

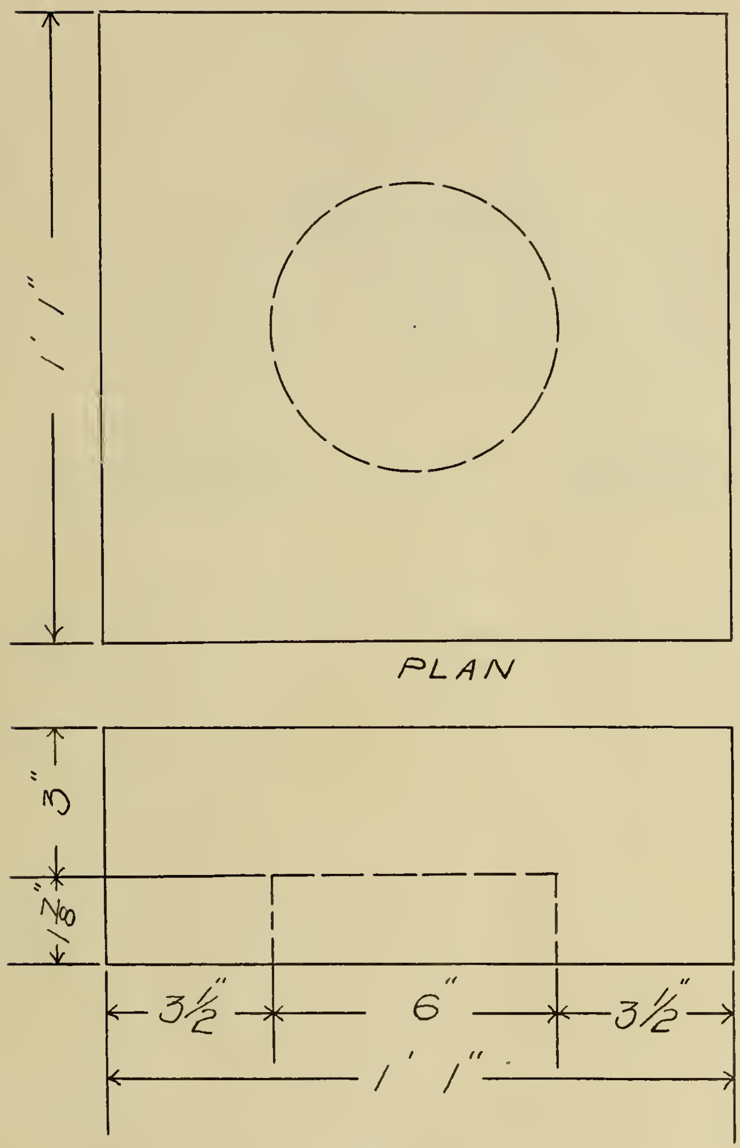
The specimen decided upon is shown in Plate I Page 6. It consists of a concrete block 13 inches square by 4-5/8 inches thick, with a cylindrical hole at the center 5-7/8 inches in diameter and 1-5/8 inches deep, leaving an effective thickness at the center of (4-5/8-1-5/8) 3 inches. This specimen will be referred to hereafter as Class A.

The reason for having the body of the block thicker that the actual shearing portion was to give a large area to resist any horizontal tensile stresses induced by the vertical compression from the applied load, and hence less liability to failure by tension. It was also thought that by using plaster of paris to obtain an even bearing for load and for base of block, it would be possible to practically do away with any possibility of failure by bending.

In order, however, to test the assumption that the larger area of the main block would lessen the liability of failure, is was decided to make a number of specimens such as are shown in Plate II Page 7. This test piece consists simply of a block 13 inches square by 3 inches deep, to be tested in same way as the block first described. This specimen will be called Class B.

PLATE I

Test Specimen Class - A -

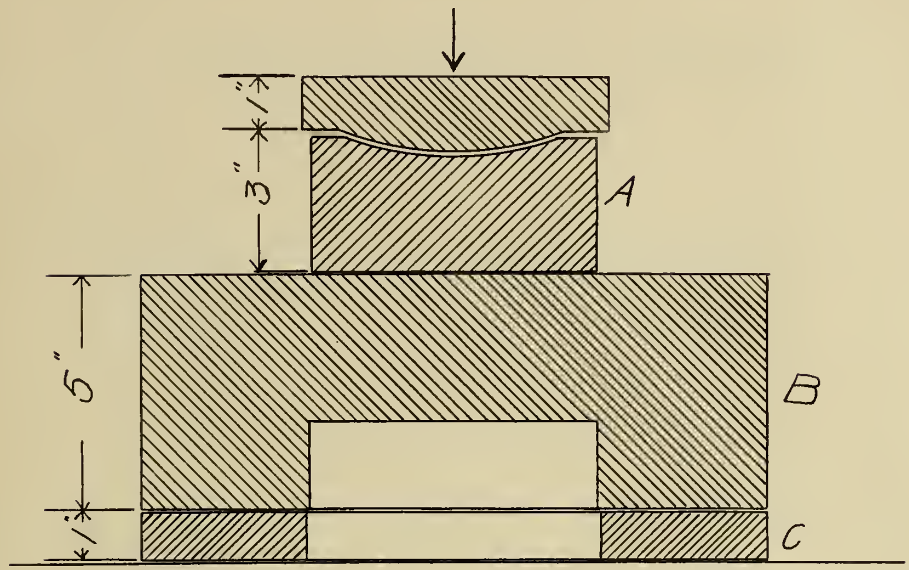


ELEVATION

PLATE 2

Method of Applying Load.

Fig. 1



Test Specimen Class - B -

Fig. 2

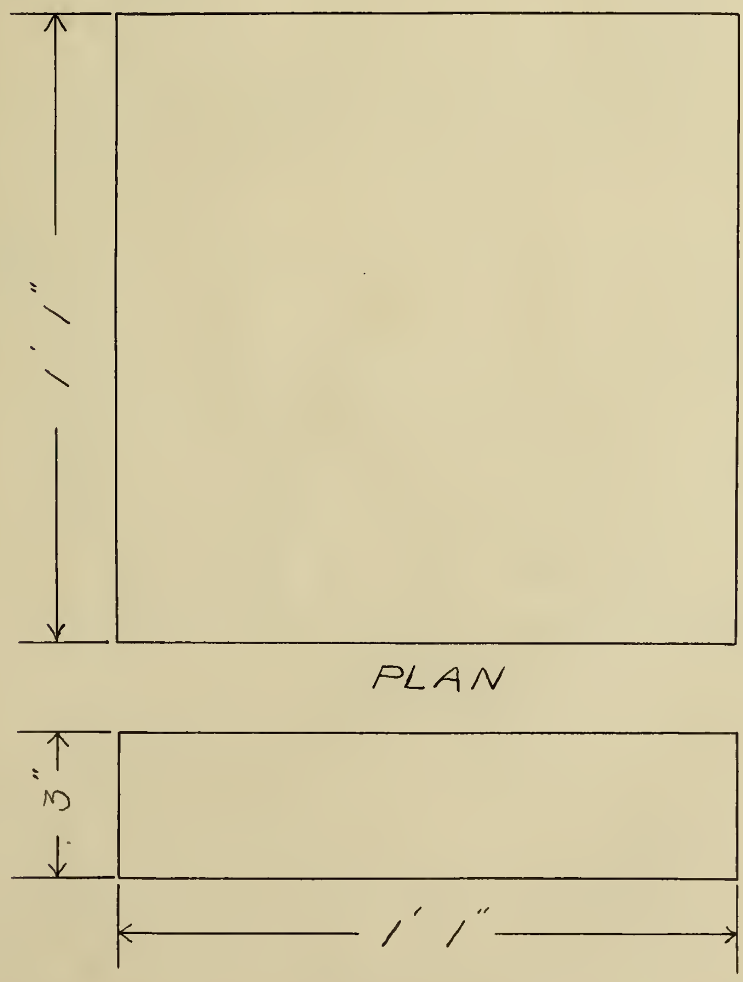
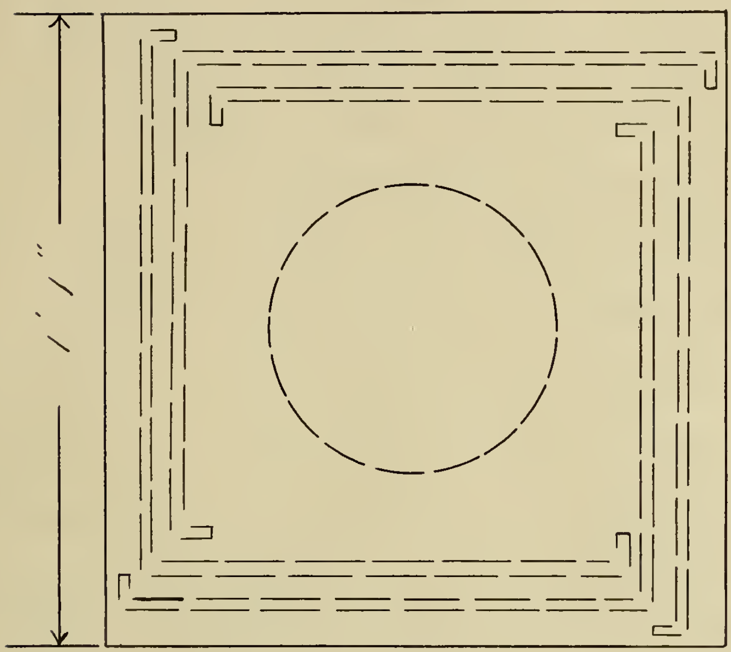
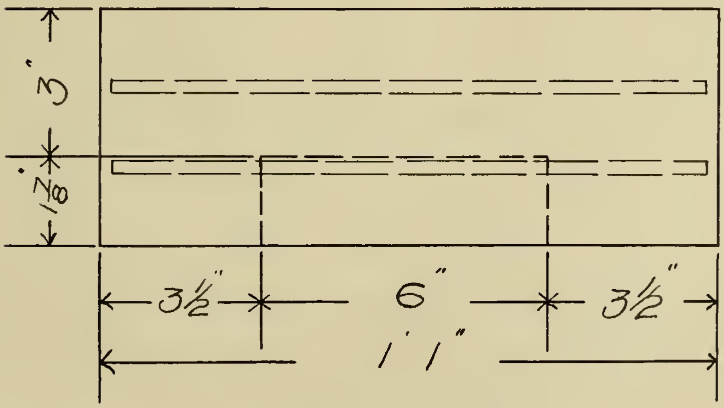


PLATE 4

REINFORCED TEST SPECIMEN



PLAN



ELEVATION

Several preliminary tests made upon the two forms of blocks just described, at an age of about 3 weeks, showed that before the cylindrical section could be sheared out, the specimen cracked. Fig. 1. The central portion of the block, being under great com pression, tended to spread out. This tendency was resisted by the surrounding concrete, and caused "induced tension" at right angles to the direction of application of load. To meet this difficulty some reinforced test pieces were made by reinforcement bars being placed so as to provide for the induced tensile stresses. Two blocks were reinforced with one-fourth inch twisted square rods as shown in Plate IV, Page 8 , the other two were reinforced with flat bars 1-3/8 inches by 3/16 inches, illustrated in Plate V, Page 10. The reinforced specimen will be known as Class C.

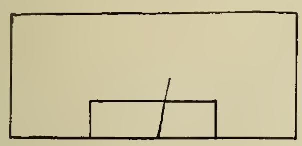

Elevation

Fig. 1

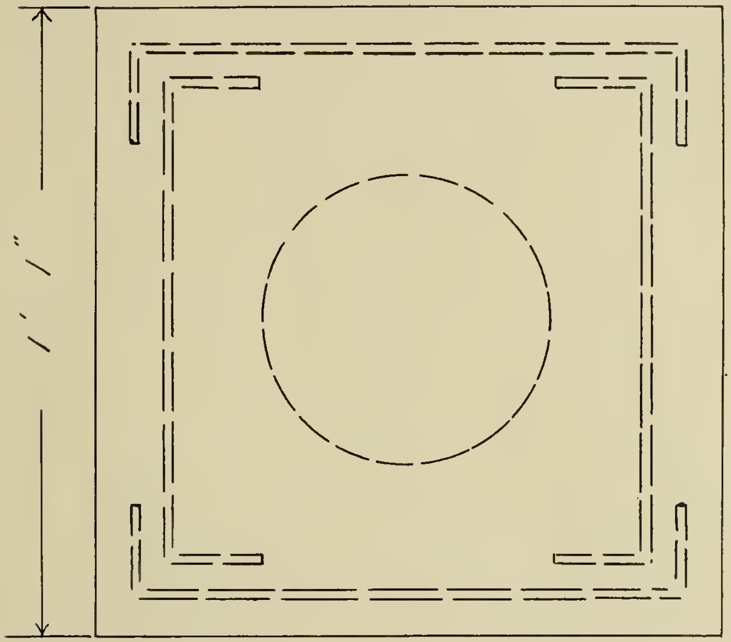
Showing Position of cracks.

The forms used in making test pieces for Classes A and C are shown in Plate III, Page 11. The base platform was 18 inches square and consisted of 6" x 3/4" planking nailed to 2" x 2" cross pieces. To the center of this platform was nailed a cylindrical block 5-7/8" in diameter and 1-5/8" thick. The sides consisted of 4-5/8" x 3/4" planking which could be bolted together and clamped to the base platform, the inside dimensions of the form being 13" x 13".

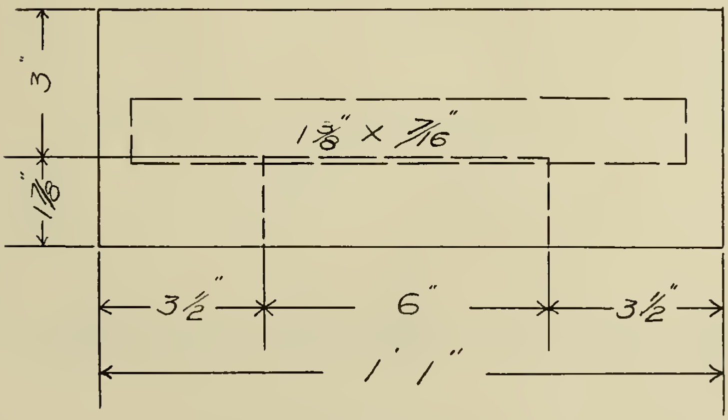
In making these specimens, considerable difficulty was encountered, because of the fact that owing to the contraction of the concrete, and expansion of the wood, due to absorption of water, it was impossible to remove the forms without destroying the cylindrical block and causing injury to the concrete. To meet

PLATE 5

REINFORCED TEST SPECIMEN



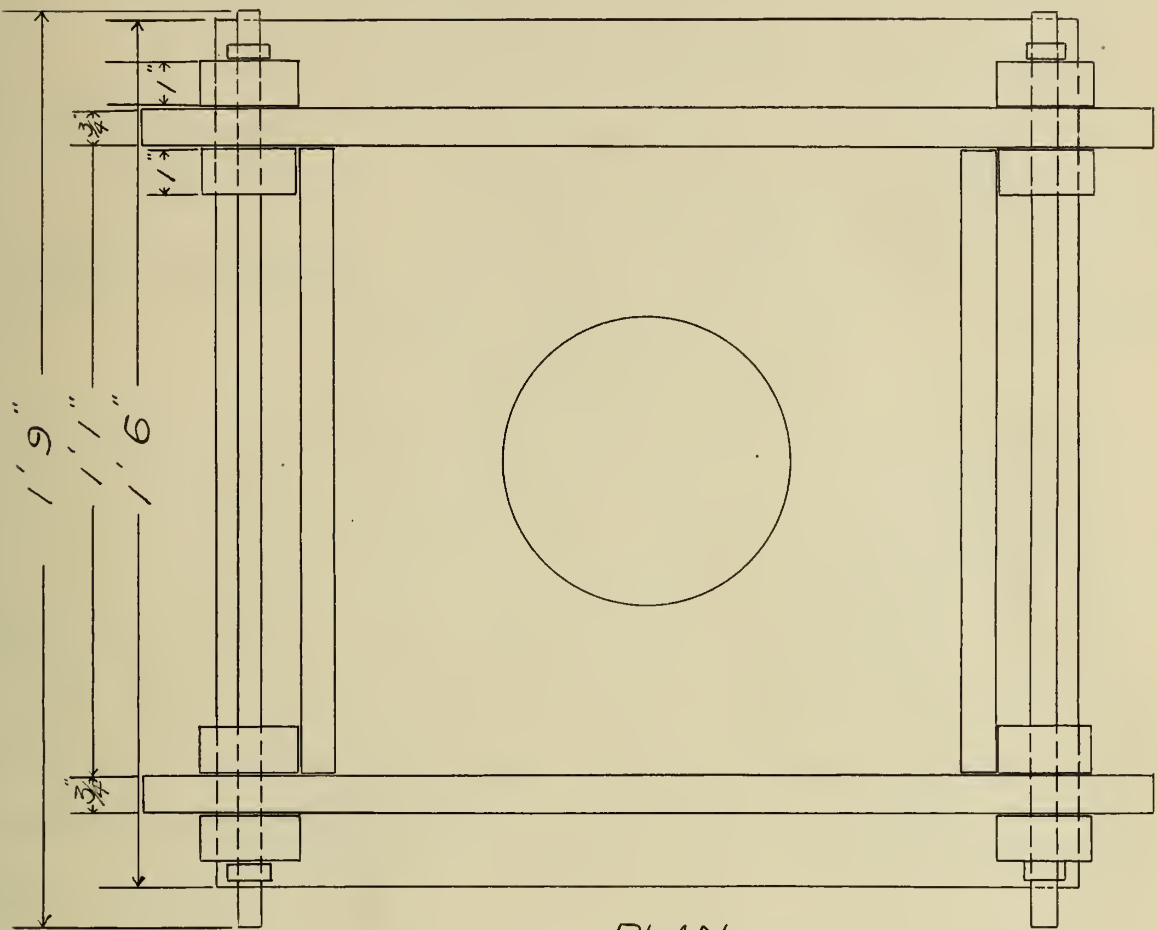
PLAN



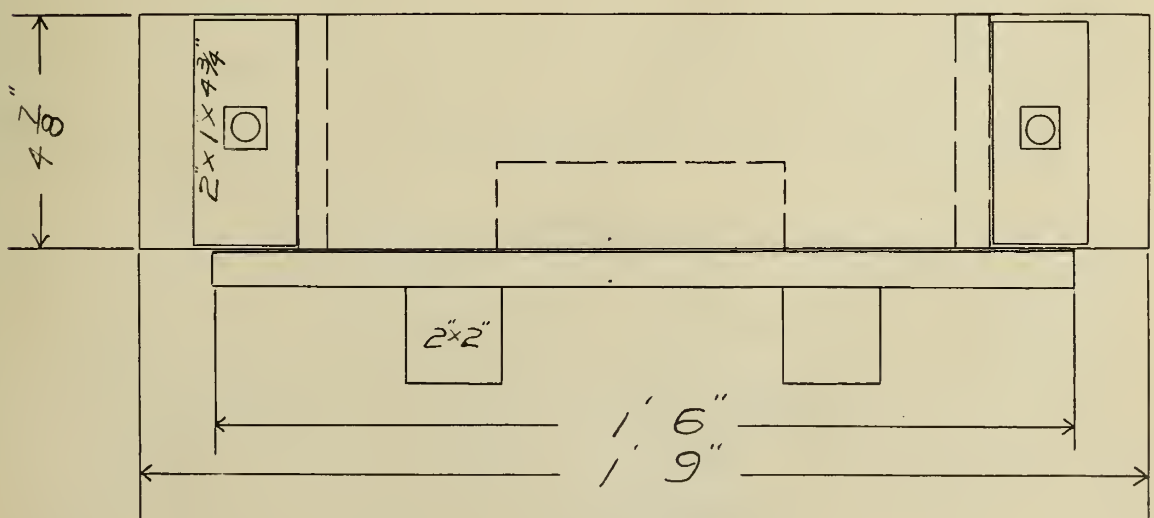
ELEVATION

PLATE. 3.

FORMS FOR MAKING CONCRETE BLOCKS.



PLAN



ELEVATION

this condition of affairs, tapering blocks, as shown in Fig. 2, were substituted for the right cylindrical pieces. Care was taken to sand paper and oil them thoroughly and although this plan did not entirely overcome the difficulty, it helped considerably. On Plate VI, Page 13 is given the design of forms for such specimens as would be recommended for further tests on this subject.

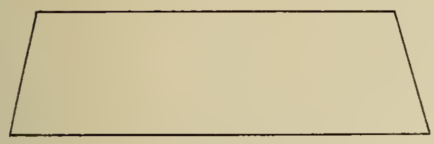


Fig. 2

Tapering Block

The forms of the specimens of the second class consisted simply of 4-3" x 3/4" planks which could be bolted together, forming a 13" x 13" box. No base platform was used, the form being placed on a sheet of building paper laid directly on the floor.

The blocks were all made of 1:3:6 concrete referred to under "Description of Materials" page 4 , and were made in two layers, thoroughly tamped, the upper surface being finished with a thin plastic mortar.

It was desired to determine relative shearing strengths of concrete which had set in air, and that which had been stored in water, hence part of the blocks were left in water, and part allowed to stand in the air, being sprinkled two or three times.

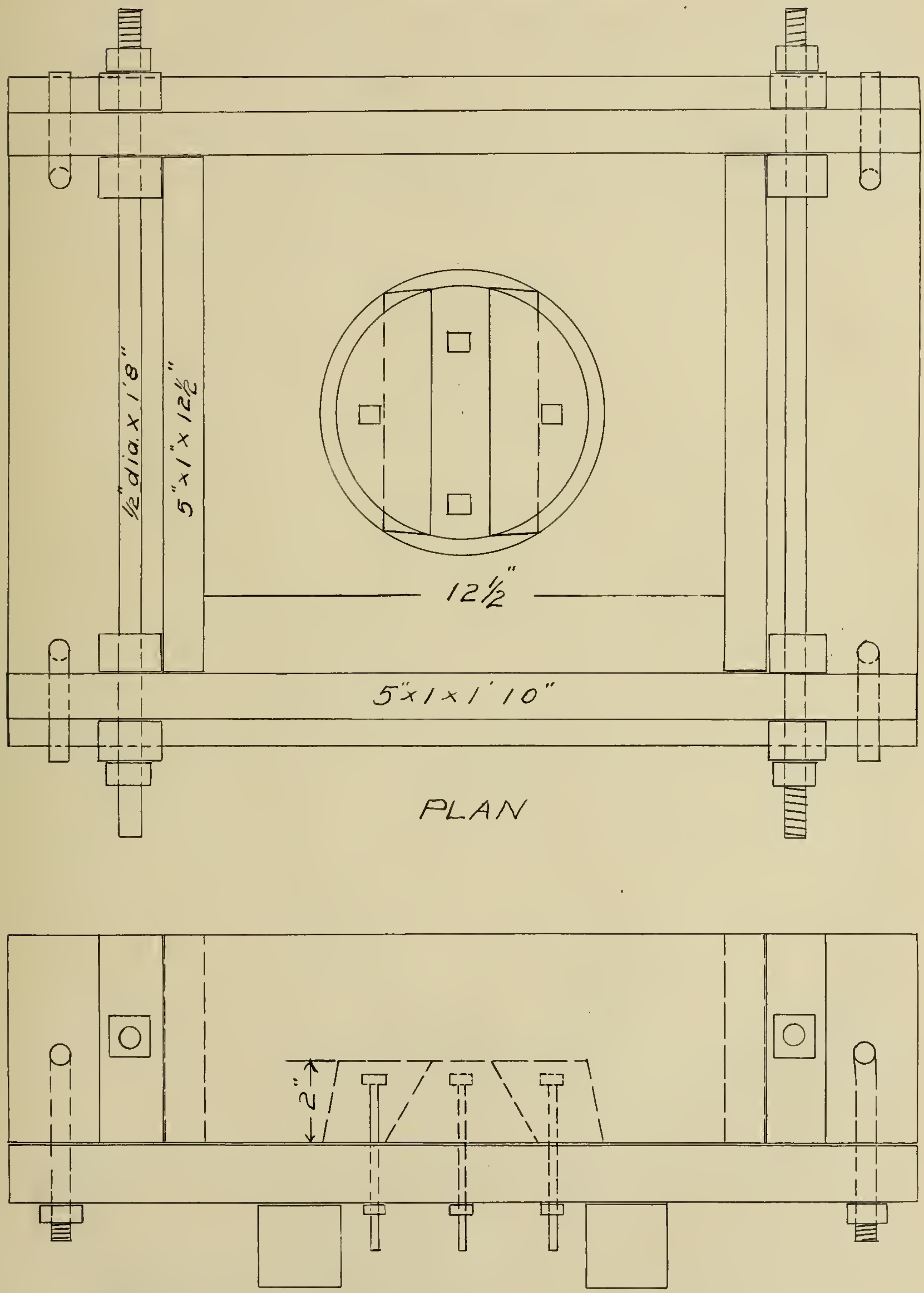
(b) Reinforced Concrete Beams.

A detailed description of the forms, method of manufacture, etc., of the reinforced concrete beams is given by Mr. R. E. Renner in his thesis, "Effect of Release of Loads", presented June, 1905.

Four beams were tested in this investigation, Numbers 14, 15, 18 and 60. All of these are of the regulation uniform size used in the tests carried on in the University of Illinois Laboratory

PLATE 6

PROPOSED FORMS FOR SPECIMENS



of Applied Mechanics in 1905, being 8 inches wide, 11 inches deep, 13 feet long. Beams 14, 15, and 18 were of 0.98 percent. reinforcement, each containing 4 one-half inch plain round bars placed 10 inches from top of beam. The proportions of the concrete in these beams were 1:3:6, measured loose and mixed as described under "Materials". Beam No. 60 has 1.1percent. reinforcement, there being 2 three-fourths inch bars 10 inches from top, the proportions of the concrete being 1:2:4.

DESCRIPTION OF TESTS.

(a) Plain Concrete Test Pieces.

The tests were made on Philadelphia 100,000 pound testing machine. Loads were applied continuously up to the ultimate load, first appearance of cracks and ultimate load being noted.

The method of applying the load is shown by the photograph Plate VII, Page 16, also by the cross-section sketch Plate II, Page 7. "A" is a spherical bearing block, through which the load is applied. "B" is the test piece. "C" is a cast iron bearing plate, 13 inches square by 1 inch thisk, with a circular hole 6 inches in diameter at the center. In adjusting the piece in the machine, plaster of paris cushions were used for both plunger and base plate, and care was taken to center the plunger directly over the hole in the concrete block and in the base plate.

(b) Reinforced Concrete Beams.

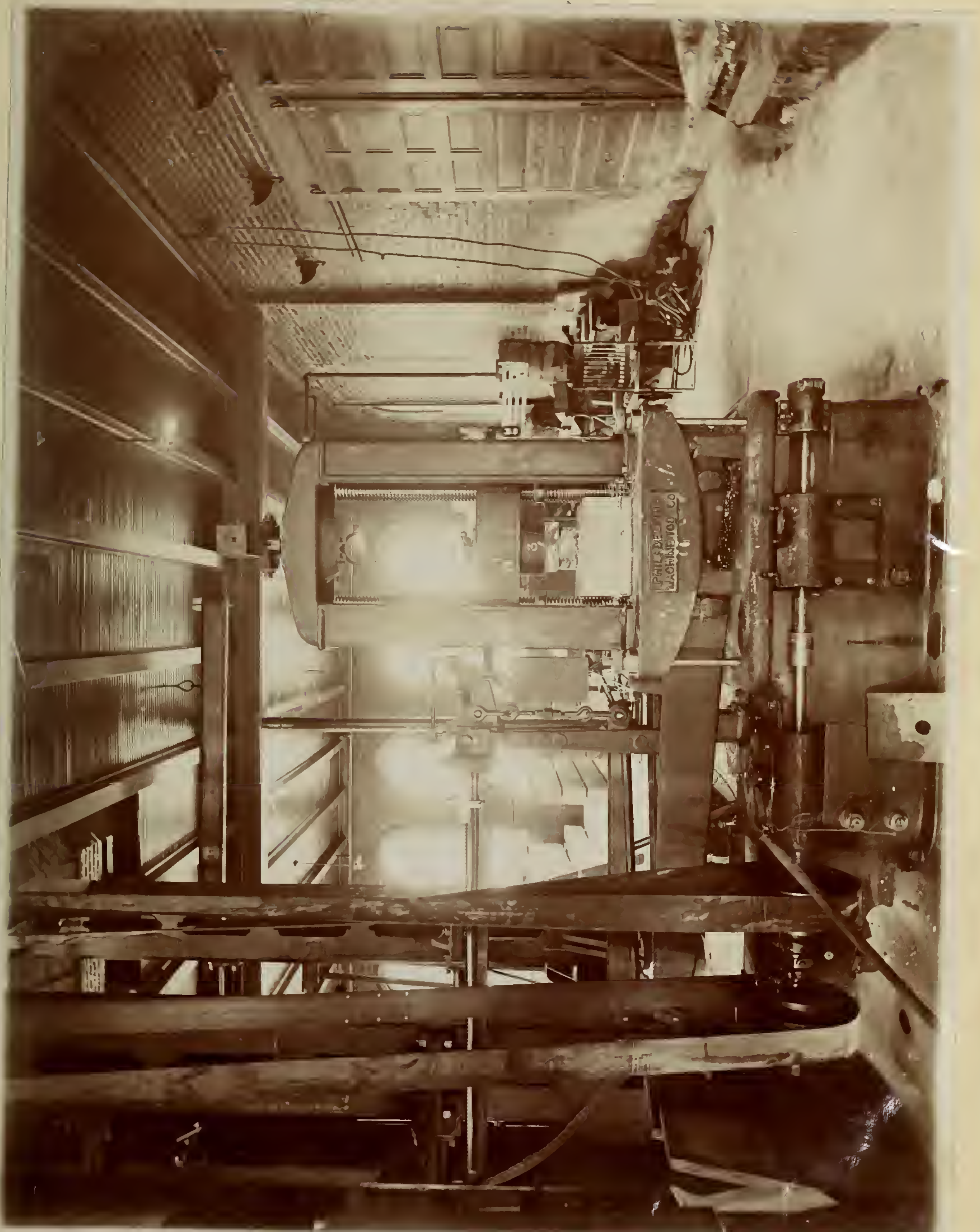
In all the tests the load was applied at two points 7-1/2 feet apart, 3 feet 9 inches each side of the center. The extensometers were placed 42 inches apart, 1 foot 9 inches each side of the center. The supports were 12 feet apart.

Loads were applied continuously from zero to ultimate load, the deflection and extensometer readings being taken for each increment of 1000 pounds.

In other particulars, the method of testing is identical with the general method described by Mr. E. T. Renner in his thesis on "Effect of Release of Loads", presented June, 1905.

PLATE 7

Specimen Ready for Test.



OBSERVED DATA.

(a) Plain Concrete Test Pieces.

Tables XI to XI, pages 38 to 41, give the general data for the concrete blocks tested for shear. There is considerable variation in the results obtained from the unreinforced specimens, but the manner of failure was the same in nearly all cases. The characteristic manner of failure was as follows: At a load of one-third to one-half of the ultimate load hair,cracks, Fig. 8, would appear

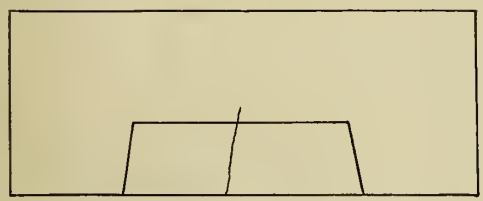


Fig. 8

Fig. 9

at the bottom and middle of the exterior of the block. At first these cracks were scarcely visible, and only extended two inches upward from the bottom, but as the load increased, these cracks would open, slowly at first and more radidly near the end, until at the ultimate load they were about one-sixteenth of a inch wide at the bottom, and extended to the top of the block, Fig. 9, where they were much finer. The blocks could then be broken apart with the hands, leaving the punched circular piece in the center and the four quarters on the outside. In all instances the punched piece had cracks on the bottom, which in most cases would only extend one-half to two-thirds the distance to the top. However, in a few cases this part of the specimen would also be broken. Plate VIII, page 18, plainly shows the manner in which the block breaks into its several parts.

Another way in which specimens sometimes failed was by the whole block breaking into two parts, as a beam does, while in still another the corners rose a small distance much as a plate does when it is punched.

The method of failure of the three reinforced specimens Nos. 49, 50, and 51 differed from that just described in that the cracks appeared relatively later, and had no openings when the ultimate load was reached. Test piece No. 48 cracked badly when tested, the cracks running out from the center of the blocks toward two of the corners.

(b) Reinforced Concrete Beams.

The measured values for the beam tests, consisting of the loads, extensometer readings, and deflections, are given in Tables 13 to 16 , pages 41 to 45. The general behavior of the beams during the tests was carefully followed, all cracks, etc., being noted.

BEAM No. 14. This beam was cracked entirely through in three places, due to an accident a few days previous to the test, two of these cracks being about 2 feet on either side of the center respectively, the third being about 3 feet from one end. As the load was applied, the original cracks opened slightly, but no new ones were developed until at a load of 7000 pounds. Then two fine vertical cracks appeared at the bottom of the beam, one under a load point, the other at the center. At 9000 pounds six small vertical cracks about 1-1/2 inches to 2-1/2 inches long had appeared at the bottom and near the ends of the beam. These cracks gradually became longer until at 12,000 pounds they extended to the middle of the beam. At this load a second vertical crack about three inches

long had appeared directly opposite the one first noticed at the center, which at this load extended to the middle of the beam. At 13,000 pounds about 10 small cracks 2 to 3 inches long were visible. At 15,000 pounds very marked cracks appeared on both sides and about 11 inches from support. These cracks extended diagonally upward toward the support at about an angle of 30% with the vertical until it reached a point near the neutral axis. They then took a larger angle and came out under the load points. At 15,400 pounds the beam broke very suddenly through these cracks. The end of the beam was completely severed from the rest, and the portion of the concrete in the end below the steel separated from that above.

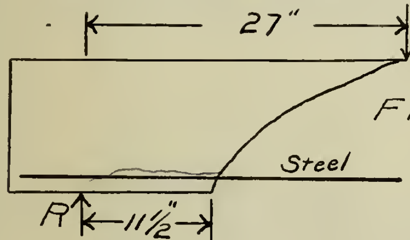


Fig. 10.

Ruptured End of Beam

The photograph on Plate IX shows very clearly the nature of the break as the beam appeared in the machine.# Fig. 10 shows the dimensions of the ruptured section.

BEAM No. 15. In testing this beam, no cracks were noticed until a load of 14,000 pounds had been reached, at which time four fine vertical cracks appeared near the loads running from the bottom to within 4 or 5 inches of the top. At 16,000 pounds, diagonal cracks similar to rupture line in No. 14 appeared on both sides of each end. At 17,000 pounds, the cracks near one end had lengthened somewhat, and at 17,600 pounds, with the machine at rest, the beam broke very suddenly in almost exactly the same way as did No. 14.

BEAM No. 18. This beam was cracked slightly about 2-1/2 feet to one side of the center. The crack was too small, however, to impair the strength of the beam to any great extent. At 10,000 pounds, the original crack had opened somewhat at bottom, one slight

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Plate IX

BLUEPRINT SHOWING DIAGONAL CRACK

vertical crack about 1-1/2 inches long had appeared under one load, and four other small vertical cracks from 2 to 5 inches long were seen. At 12,000 pounds these cracks had lengthened slightly, but no new ones had appeared. At 13,000 pounds, a diagonal crack, about 6 inches long and about 30° with vertical, had appeared on both sides near one end. At 15,000 pounds two more diagonal cracks were noticed, running out from bottom on 60° line toward load, these cracks being above the others, as shown in Fig. 11. At 18,000 pounds, the diagonal cracks last mentioned had extended to within 3 inches of the top and had opened considerably. The beam broke suddenly at 18,800 pounds, the method of failure being identical with Nos. 14 and 15.

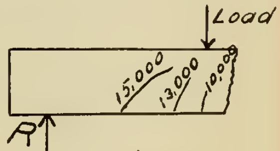


Fig. 11.

BEAM No. 60. In this beam the cracks appeared under a smaller load than in the other three. At 6000 pounds a small vertical crack was visible near the center of the beam, and at 8000 pounds, four more appeared, two on each side of the beam, and about 6 inches inside the load points. At 9000 pounds the cracks had lengthened, and were about 7 inches long. There also appeared 3 or 4 little cracks. When 12,000 pounds was reached, the small cracks lengthened to about 4 inches, and the large cracks near one end had opened somewhat but not lengthened. At 13,200 pounds the scale beam remained balanced until the beam deflected 0.54 inches. The load then dropped down to 12,700 pounds, where it varied a couple hundred pounds, the deflection reaching 1.08 inches. The crack had opened up to one-fourth of an inch at the bottom and extended to an inch and half from the top when the concrete began to crush on top. The test was stopped after having applied the load for 12 minutes. The crack had spread to seven-eights of an inch at the bottom, and the concrete was badly crushed on top. The manner of failure in

this beam was by vertical cracks inside the load rather than diagonal cracks between the loads and supports.

Table II. shows a summary of the tests.

Table II

Summary of Tests

Ref No	% of Reinf't	Dist. in In. from Center		Max Load	Manner of Failure
		Load	Exten'ters		
14	.98	45	21	15,900	Diagonal Cracks
15	.98	"	"	17,600	" "
18	.98	"	"	18,800	" "
60	1.10	"	"	13,200	Vertical "

Curves showing the unit deformations, deflections, and position of neutral axis for each beam as the load increased are given on pages 34 to 38.

DISCUSSION OF RESULTS.

(a) Plain Concrete Test Pieces.

In order to determine the effect of the method of storage (curing) of the specimens upon the ultimate shearing strength of concrete, some of the blocks were allowed to set in water, and others in air. The latter were placed on elevated planks in a channel, and two or three times the water rose enough to submerge them for about a day. It might be thought that the concrete stored in water would develop a larger ultimate shearing strength, because it would set more uniformly and slowly, and hence would be less liable to crystalization. However, the conclusion is not borne out by the results given in Tables VII. and IX. Table VII. gives an average of 777 lb. per sq. in. for 11 tests on blocks setting in water, while Table IX. gives an average of 796 lb. per sq. in. for 17 air-stored blocks. This difference may be explained by the fact that specimens 1, 2, 3, 4, and 6 were made with the forms having cylindrical blocks. With these cylindrical blocks great difficulty was encountered in removing the forms and damage was probably done to the test specimens. Leaving these specimens out of consideration, the mean ultimate shearing strength of the water-stored specimens becomes 853 lb. per sq. in., a much greater value than obtained from the test pieces stored in air. A comparison of Tables VIII. and X. gives 729 lb. per sq. in. for water-stored blocks, and 679 lb. per sq. in. for those stored in air. In both cases the results of undamaged specimens setting in water were about 7% stronger than those in air.

To make a comparison of ultimate shearing strengths at differ-

ent ages, the preliminary tests were used. The shearing value at different ages can be seen in Table III. page 25.

Table III

Comparison of Unit Shear for Different Ages.

Ref No	Age Days	Class of Specimen	Storage	Unit Shear lb. per Sq. In.
38	7	B	air	447
7	19	A	water	632
5	19	A	"	440
0	26	B	"	523
Average	60	A	both	777

Test piece No. 5 was tested with the block upside down and did not give as good results as other test pieces of same age, which was probably due to the fact that the plunger fitted tightly in the depression and required a load of 1000 lb. to force it into place.

From the table it can be seen that concrete gains in shearing strength very rapidly the first few days, probably gaining two-thirds of its ultimate shearing strength within twenty days.

The general design of all specimens tested was the same, e. i., a right cylinder of concrete 5-7/8 inches in diameter, and 3 inches deep was punched out of a block. A comparison of results of the Tables VII. and IX., VIII. and X., and XI, shows that the modifications made in the test specimen have a great effect upon the ultimate shearing strength. Table XII. page 26 shows comparison of these specimens.

In comparing results it should be remembered that (see storage) in class "A" 5 specimens were damaged, and if these were left out of consideration the mean would be 800 lb. per sq. in. It is the

opinion of the investigators that class "C", for reasons stated below, gives results nearest strength.

Table XII

Effect of Shape of Specimen.—

Class	No of Specimen	Storage	Ult. Shear lb per Sq.In.
B	7	water	701
B	9	air	
A	11	water	789
A	17	air	
C	3	air	1124

As concretes vary considerable, absolute values would mean little without further tests on special kind, and so a comparison will be made with 13 compressive tests made on 6 in.-cubes of the same concrete. The results of these tests are to be found in a thesis by Mr. E. T. Renner, mentioned above, and give an average ultimate compressive strength of 1400 lb. per sq. in. at 60 days. The value found for shear, viz., 1124 lb. per sq. in. is 80% of this compressive strength. As far as the investigators know this is a much greater result for shear than has been reported to the public, the nearest obtainable results being by Prof. Charles M. Spofford (refer to introduction) who from experiments found that the ultimate shearing strength was 50% of the compressive strength. The results which are perhaps furthest off, are those by M. Christophe, M. Feret, and M. Considere. Their results together with the Prussia and New York Building Regulations are given on pages 1 and 2.

The general method of failure in the unreinforced specimens, as already described on page 17, consisted in the cracking of the

block at the botton from the center to the middle of the sides. The cracks were caused, for reasons explained below, by tension at right angles to the downward pressure, and it was thought at the time that these cracks might possibly have weakened the ultimate shearing strength, but no definite conclusion on this point was reached until the reinforced specimens had been tested.

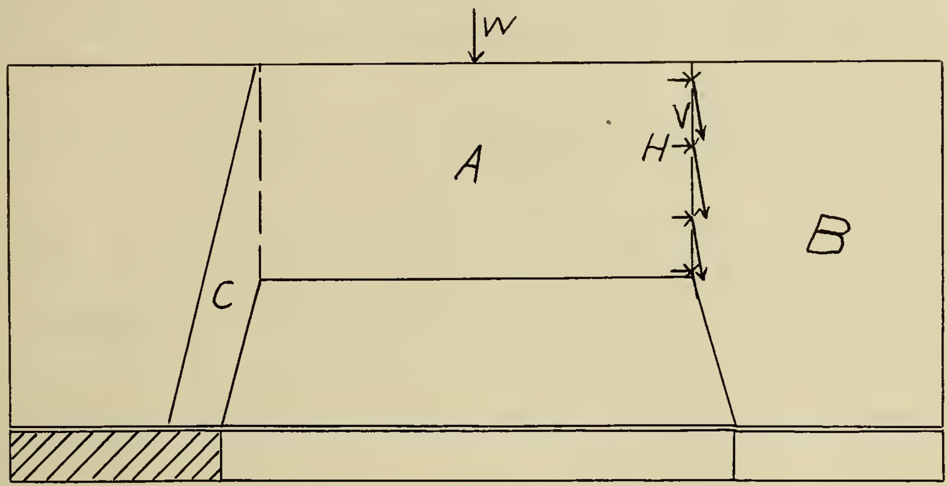


Fig. 12.

Fig. 12 is a vertical section through the center of a class "A" specimen and will be used in explaining why the so called tension cracks appear in the specimen. As the force W is applied, it causes the vertical shearing forces V, and also the horizontal forces H, due to the lateral expansion of "A", to act in the manner shown. These two forces combined making a series of parallel forceswhich compress the material in B, causing tensile forces perpendicular to them. In some of the experiments it was noticed that the material in "C" would hold to the part "A" when the block was broken apart, making a frustrum of a cone with slope of about 1:4, but that this extra material could be easily broken off.

By an inspection of Table XII. page 26 it is seen that the unreinforced specimens have an ultimate shearing strength of

789 lb. per sq. in. against 1124 lb. per sq. in for reinforced specimens, the latter being nearly 50% greater than the former, seems to indicate that the plain concrete specimens were effected to some degree by the tensile cracks, even though there was no perceptible drop in the scale beam when they first appeared.

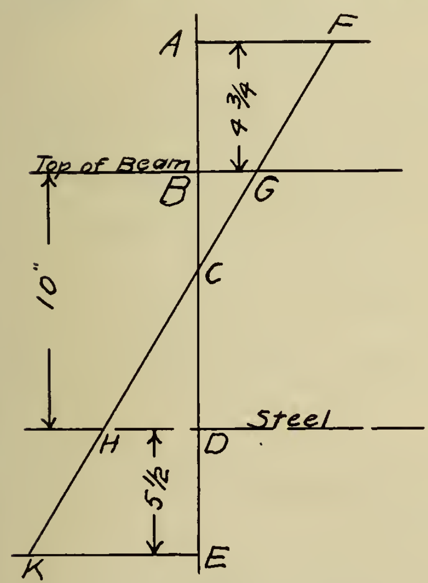
As neither the plain nor reinforced specimens were entirely satisfactory, a new size of test piece and different reinforcement is here proposed for the tests on this subject next year. Plate 6 page 13 gives a design of forms which it is believed will give a very satisfactory specimen. For reinforcement two steel hoops are recommended instead of straight bars. The bars were able to resist the tension at the quarter points of the blocks, but could not at the corners. It is thought that these hoops will resist the tension all around the block, and hence be more satisfactory.

(b) Reinforced Concrete Beam.

The results of the test were all worked up in the same way with the exception of No. 14. In this beam a crack of consider-

able size already existed in the top of the beam between extensometers, and as the load was applied the crack closed up causing the upper extensometers to move too rapidly. An attempt was made to allow for this in the computations, but without complete success, as can be seen from an inspection of the curves for this beam. In the table for beam No. 14, Page 42, are given the deformations figured from observed data and in parenthesis, deformations figured after some allowance had been made for the closing of the crack. In beams Nos. 15, 18 and 60 no such trouble was encountered, and their respective curves are very uniform.

The unit deformation of steel and upper fiber of concrete, also the position of neutral axis were determined graphically. Extensometers Nos. 1 and 3 were 4-3/4 inches above the top of the beam; Nos. 2 and 4 were 5-1/2 inches below the steel; and steel and top of beam were 10 inches apart.



AF represents the average linear motion of extensometers Nos. 1 and 3. KE in the average of Nos. 2 and 4. The straight line FK is drawn then BG represents to scale the compressive deformation of top fiber, and HD the tensile deformation of steel, while C represents the position of the neutral axis in the beam. This is based on the assumption that a plane section before bending remains a plane section sfter bending. These total deformations, divided by distance between extensometers (viz. 42 inches) give the unit deformations, which are given in the tables.

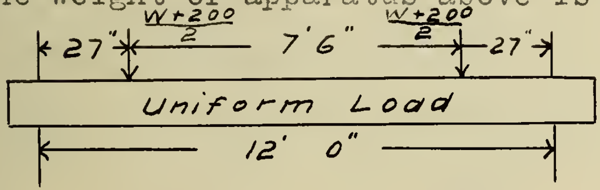
Several equations are required in computation of results and will be developed here.

(1) Distance from Steel to Center of Gravity of Compressive Forces.

It is assumed that the unit compressive stress in the concrete above the neutral axis varies as the three-fourths power of the distance from that axis, it is found by integration that the center of gravity of the compressive stresses is four-elevenths m below the top, m being the distance from top to neutral axis. If d be called the distance of steel from top, then the distance from steel to center of gravity of compressive forces = (d-4/11m).

(2) Bending Moment in Beam.

The uniform load of the beam itself is 100 pounds per foot. The weight of apparatus above is 200 pounds. The concentrated load, applied in two points 7-1/2 feet apart will be called W. The moments at the center, and at the load point are required.



$$M_c = \frac{1}{8}\cdot 100\cdot 12\cdot 144 + \frac{W+200}{2}\times 27$$

$$M_L = \left(\frac{100\cdot 12}{2} + \frac{W+200}{2}\right)27 - \frac{100\cdot \overline{2.25}^2\cdot 12}{2}$$

(3) Total Vertical Shear.

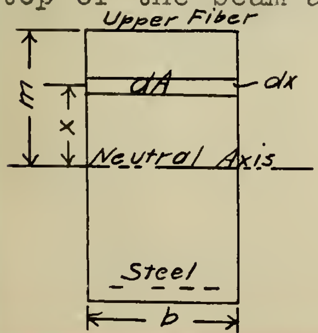
At center of beam shear = 0

At load point the total shear = W/2 + 200/2 + 1200/2 - 225.

$$V_L = 1/2(W + 950)$$

(4) Horizontal Unit Shear.

As has already been stated the compressive stresses in the top of the beam are assumed to vary as the three-fourths power of the distance from the neutral axis.



Let S = stress intensity of upper fiber.

Then $S/m^{3/4}$ = stress intensity at unit distance from neutral axis.

$S/m^{3/4} \cdot x^{3/4}$ = stress intensity at distance from neutral axis.

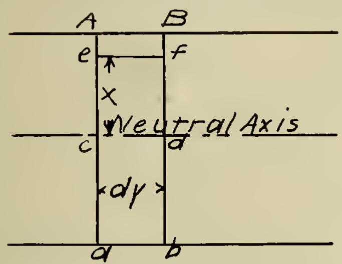
Total stress on dA = $S(x/m)^{3/4}$ b dx.

Total stress on beam above neutral axis = $\int_0^m S(x/m)^{3/4}\, b\, dx = 4/7\ Sbm$

As has already been stated, the centroid of the compressive stress is $(4/11)m$ below the top. The total moment of this stress about the steel is found to be

(I) $M = 4/7\ Sbm(d-(4/11)m) = KS$, where $K = 4/7bm(d-4/11m)$

This is also equivalent to the bending moment, in the beam.



Let Aa and Bb represent two sections at a distance dy apart.

Let M = moment at A

Let M + dM = moment at B

From (I) intensity of stress at A = S = M/K

Since stresses vary as three-fourths power from neutral axis

Stress at C = $M/K\ (x/m)^{3/4}$

In the same manner, the stress at f is found to be equal to

$$\frac{M + dM}{K}(x/m)^{3/4}$$

Total stress on surface Ae = $M/K \int_x^m (x/m)^{3/4}\, b\, dx$

" " " " Bf = $\frac{M + dM}{K}\int_x^m (x/m)^{3/4}\, b\, dx$

The difference between these two stresses in the amount of horizontal shear in plane ef.

This difference = $dM/K \int_x^m (x/m)^{3/4}\, b\, dx$

The area of ef = b dx. Therefore the unit horizontal stress = $dM/K \cdot b \cdot dx \int_x^m (x/m)^{3/4}\, b\, dx$.

Since $dM/dx = V$, the unit horizontal stress, which will be called H, equals, $V/Km^{3/4}\ (4/7m^{7/4} - 4/7x^{7/4})$ (II)

At the neutral axis x = 0, and this equation becomes $H = V/K \cdot 4/7m = 4Vm/7K$ Substituting the value of K $H = V/b(d - 4/11m)$ (III)

Equation (II) gives the horizontal unit shear at any distance "x" above the neutral axis.

Equation (III) gives the horizontal shear at neutral axis, and for all points between neutral axis and steel.

These equations also give vertical shear, as vertical unit at any point equals horizontal unit shear at same point.

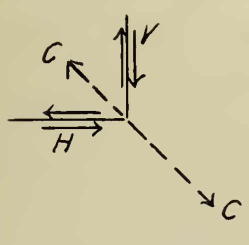
(5) Bond.

The steel is prevented from slipping by the bond between it and the concrete. Is is desired to determine this resistance between the load and the support. There is no stress in the steel over the support, while at the load there is a certain amount which will be called "T". This is transferred to the concrete in the distance between support and load, and this transference of stress is considered to be uniform between these two points. There are four one-half inch rods and the distance is 27 inches.

$$\text{Bond} = T/(\pi \cdot 1/2)\cdot 4 \cdot 27 = T/169.7$$

(6) Tension.

The horizontal shear and vertical shear combine to produce tension along a 45° line, as shown. $H = V$, $C = \sqrt{2}\,H$. But



since the area over which this stress is distributed is also multiplied by $\sqrt{2}$, the tensile unit stress at any point equals the horizontal and vertical unit stresses.

The six formulas deduced above are summarized here:

(1) Distance of steel from center of gravity of compressive forces = d - 4/11m

(2) $M_c = 13.5W + 24300$ $M_L = 13.5W + 15860$

(3) $V_L = 1/2(W + 950)$

(4) $H = \dfrac{V}{b(d - 4/11m)}$ where V = total vertical shear

(5) $\text{Bond} = \dfrac{T}{169.7}$

(6) Unit Tension in Concrete = H

Using above formulas, Tables IV. and V. page 32c were prepared from the data obtained in tests. Table IV. contains the results of the beams tested for this investigation; a discussion of the manner of failure of these can be found under "Observed Data". In order to furnish further comparison, Table V. was computed from the tests conducted by Mr. J. C. Gilmour, for his thesis entitled, "Tests on Reinforced Beams——Effect of Varying the Per cent. of Reinforcement," presented June, 1905.
The blue print on Plate LX. page 21, shows the manner of failure of these beams. From an inspection of the beam after the concrete had dropped from the bottom, it would be seen that the rods had slipped, and hence horizontal shear was not the cause of this particular failure.

Some attempt will now be made to reach a conclusion about the diagonal cracks, which appear at the bottom of the beam between the end and support, and run to the load point. It has already been stated in the introduction, that the method of failure has been spoken of as due to shear and also to tension. However, it seems that beams could not possibly have failed by shear, when the maximum shearing stress developed in any of the beams was only a 150 lb. per sq. in. as compared to the shearing strength of 1124 lb. per sq. in. which was obtained in this investigation.

There is no doubt but that beam No. 60 failed by the bars slipping, as the bond between a high steel bar and concrete would

Table IV

RESULTS OF BEAM TESTS

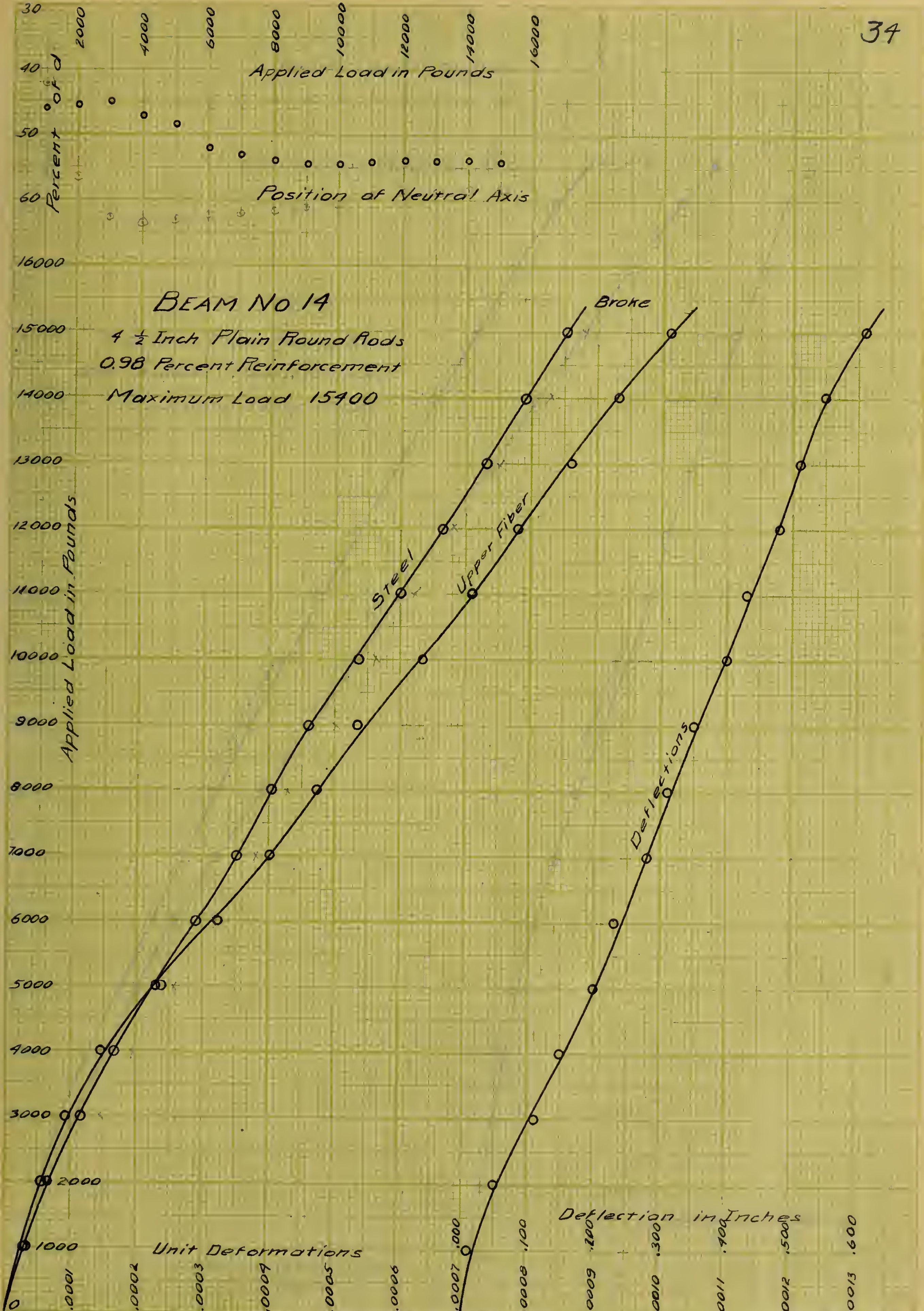
Ref No	Load	Mom. in In-lbs.		d-⅞m	Pounds per Sq. In.						
					Center		Load		Bond	Hor. Shear	Diag. Tension
		Center	Load		Total St.	Unit St	Total St.	Unit St.			
14	15,400	228,300	222,650	8.0	28,600	36,400	27,850	35,500	164	129	129
15	17,600	261,800	256,150	8.4	31,200	39,800	30,550	38,900	180	140	140
18	18,800	278,300	272,650	8.3	33,500	42,800	32,850	41,900	194	150	150
60	13,200	200,700	194,100	8.6	23,400	26,600	22,600	25,700	178	104	104
Table V											
34		333,400	331,000	8.6	38,800	29,250	38,500	29,000	113	98	98
38		361,400	359,000	8.4	43,000	33,700	42,700	33,500	113	107	107
45		324,400	322,000	8.3	39,000	26,500	38,800	26,400	86	98	98

Beams No. 14, 15, 18, & 60 loaded 27" from end.
" " 34, 38, & 45 " 48" " "

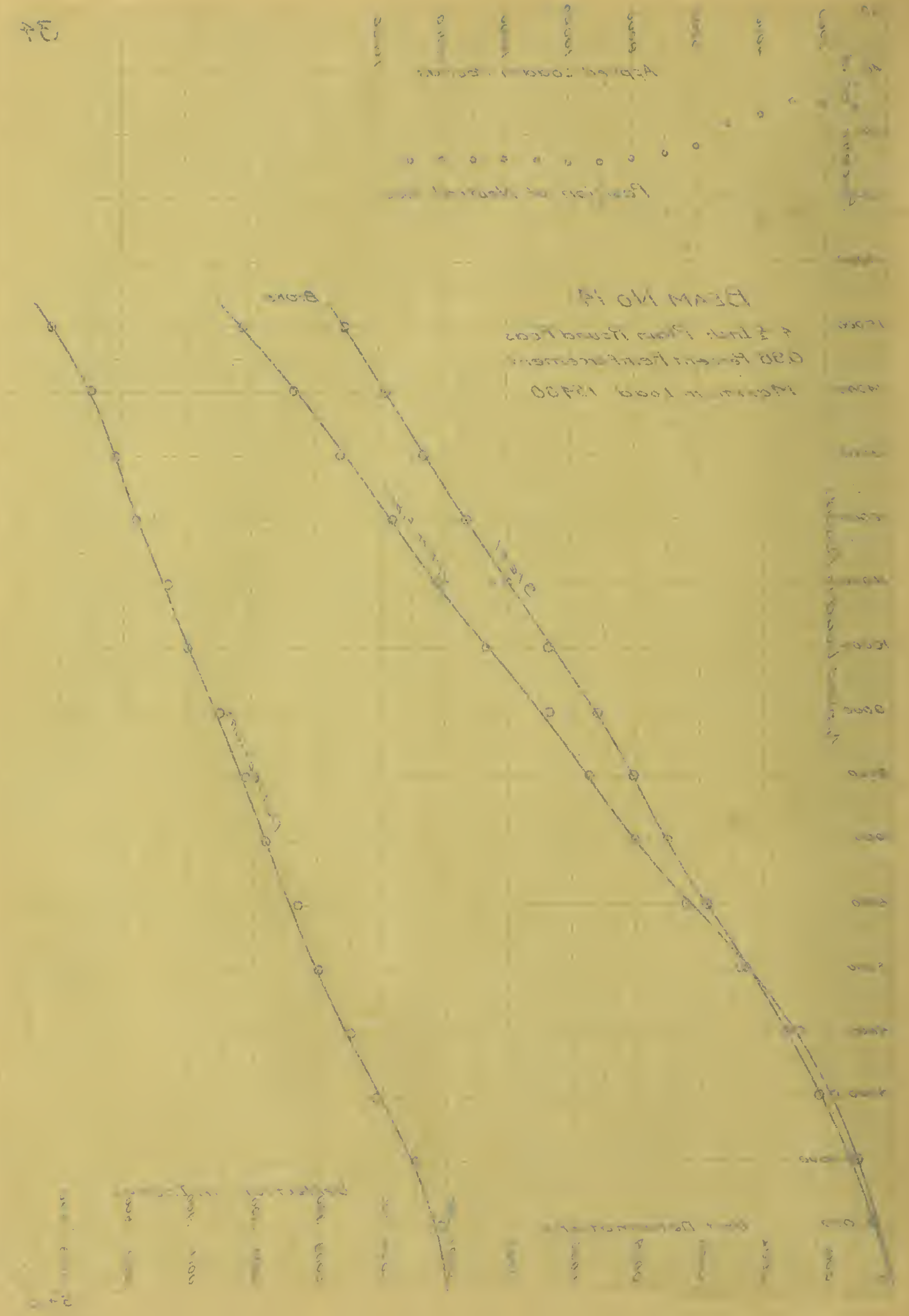
hardly be expected to exceed 178 lb. per sq. in. This beam failed by vertical cracks between the loads.

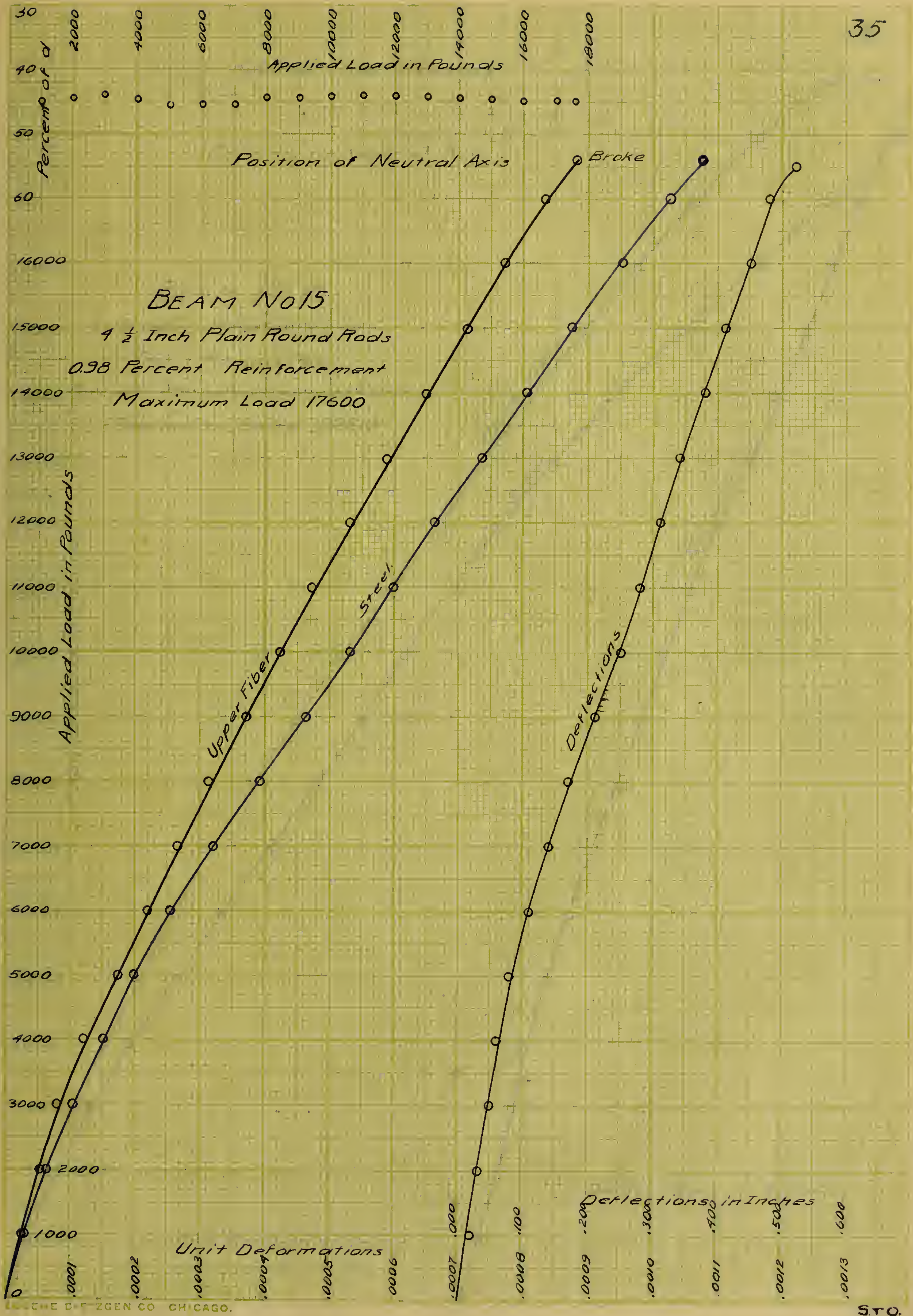
The most satisfactory explanation for the failure by the so-called diagonal cracks is reached by a study of the results for diagonal tensile stresses obtained from the use of equation 6, page 32. This tensile stress is the resultant of the horizontal and vertical shear, and would cause the failure of a beam along the 45° line provided a great enough tensile stress was obtained. The results of beams No. 14, 15 and 18 in Table IV. page 32C (the other beams having failed by the bars slipping) show that there was insufficient tensile strength in the concrete to resist the diagonal tensile stresses brought into action.

While there is not enough data to draw general conclusions, yet it seems that there is no danger of a beam breaking by actual vertical shear. On the other hand, a beam is liable to fail by diagonal cracks which have wrongly been called shear cracks, and hence careful consideration must be given to this element of the design.


Applied Load in Pounds
Percent of d
Position of Neutral Axis
Beam No 14
4 ½ Inch Plain Round Rods
0.98 Percent Reinforcement
Maximum Load 15400
Broke
Steel
Upper Fiber
Deflections
Applied Load in Pounds
Unit Deformations
Deflection in Inches
2000
4000
6000
8000
10000
12000
14000
16000
30
40
50
60
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
15000
16000
0
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
.0013
.000
.100
.200
.300
.400
.500
.600

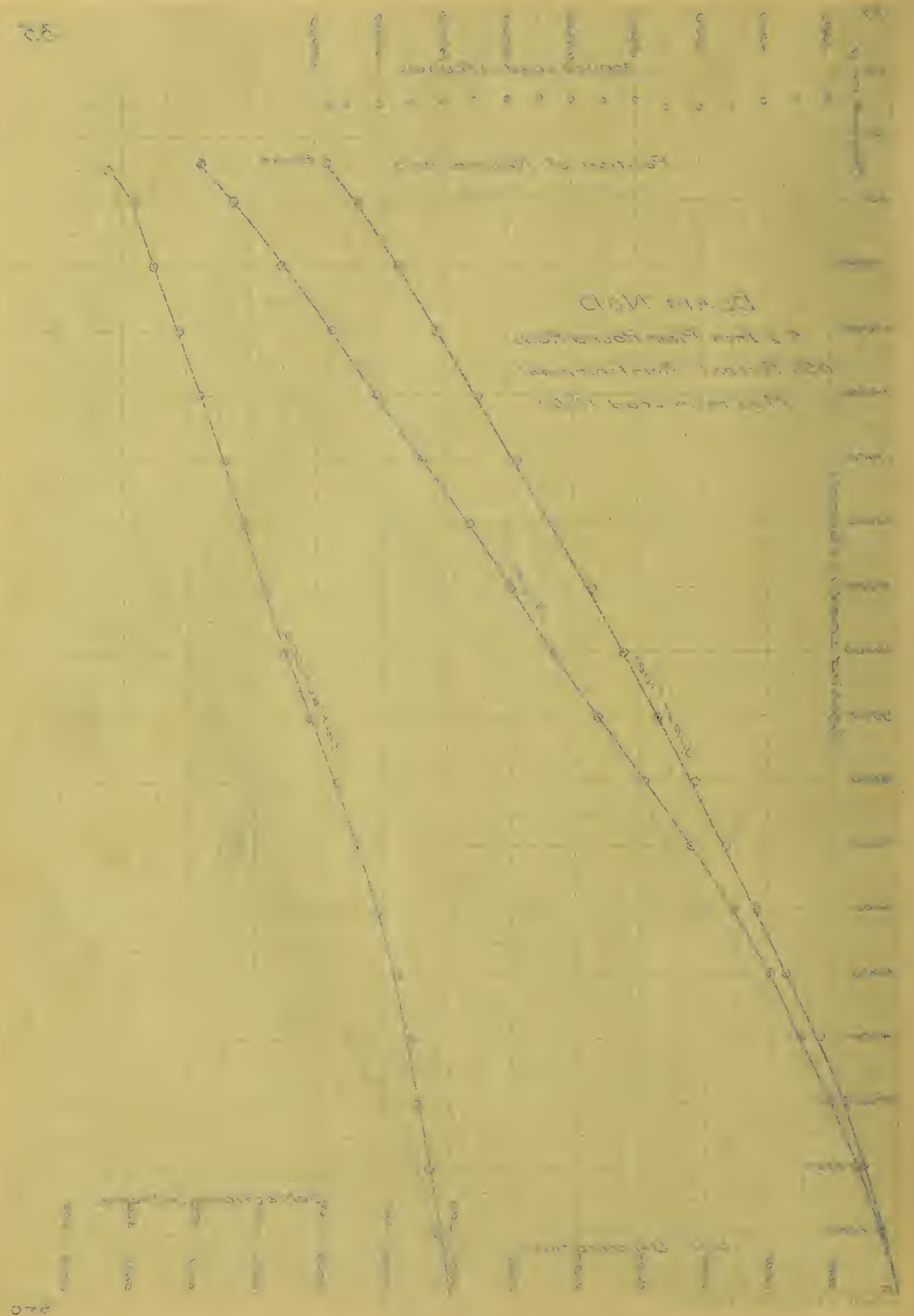
S.+O.

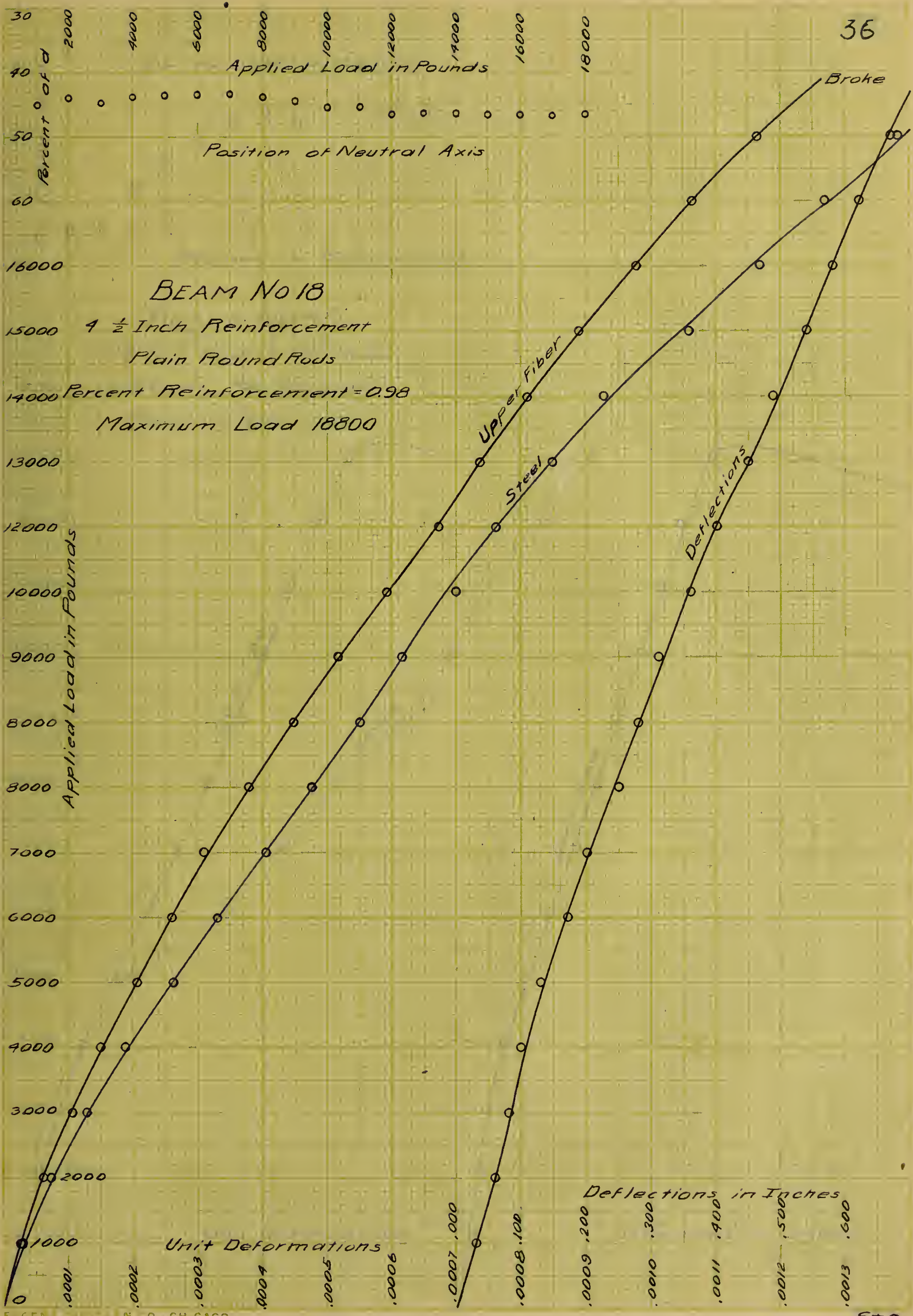

BEAM No 19
4 ½ Inch Plain Round Bars
0.90 Percent Reinforcement
Maximum Load 15750



BEAM No 15
4 ½ Inch Plain Round Rods
0.98 Percent Reinforcement
Maximum Load 17600
Applied Load in Pounds
Percent of d
Position of Neutral Axis
Broke
Upper Fiber
Steel
Deflections
Deflections in Inches
Unit Deformations
30
40
50
60
2000
4000
6000
8000
10000
12000
14000
16000
18000
1000
3000
5000
7000
9000
11000
13000
15000
0
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
.0013
.000
.100
.200
.300
.400
.500
.600

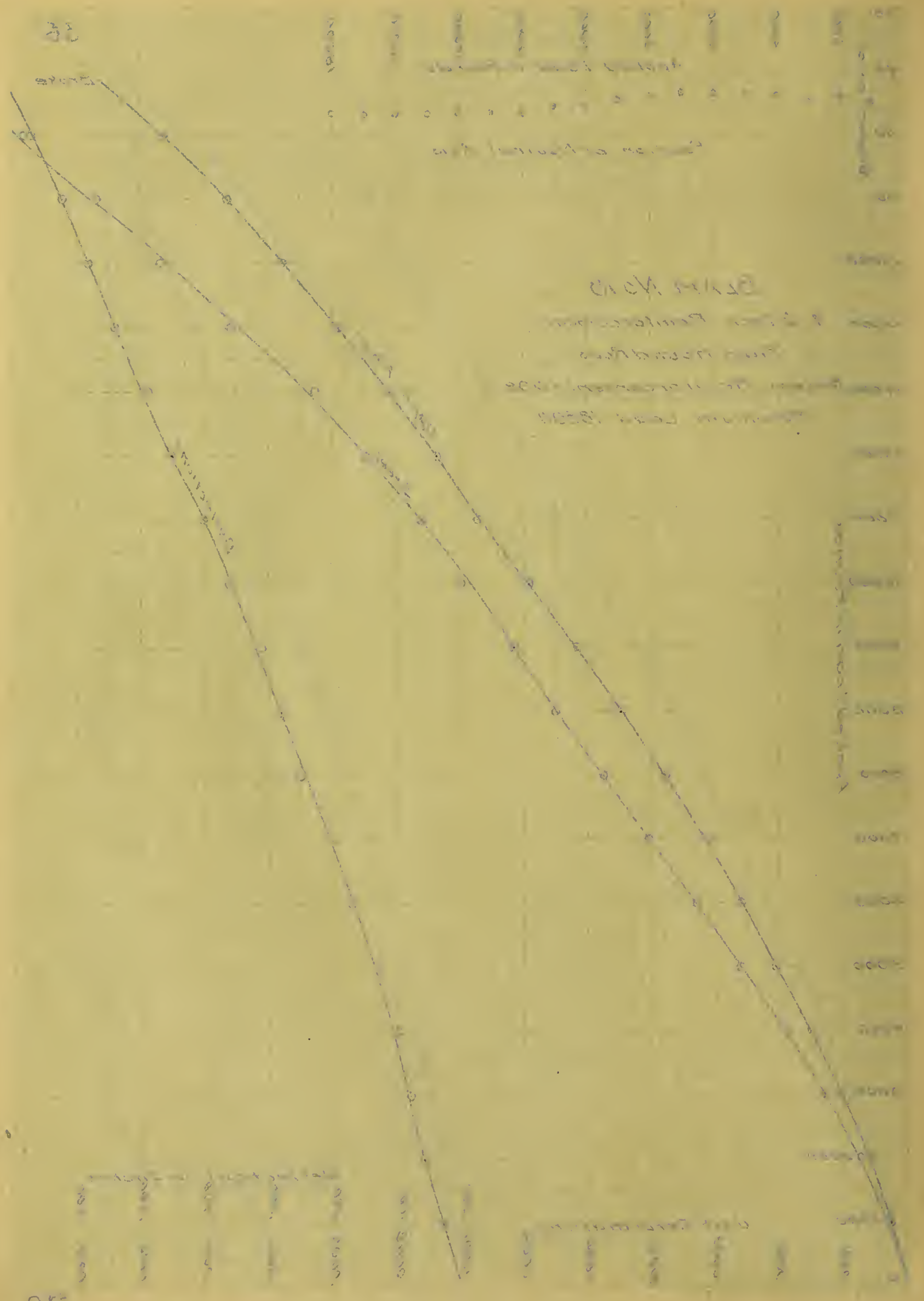
S.T.O.

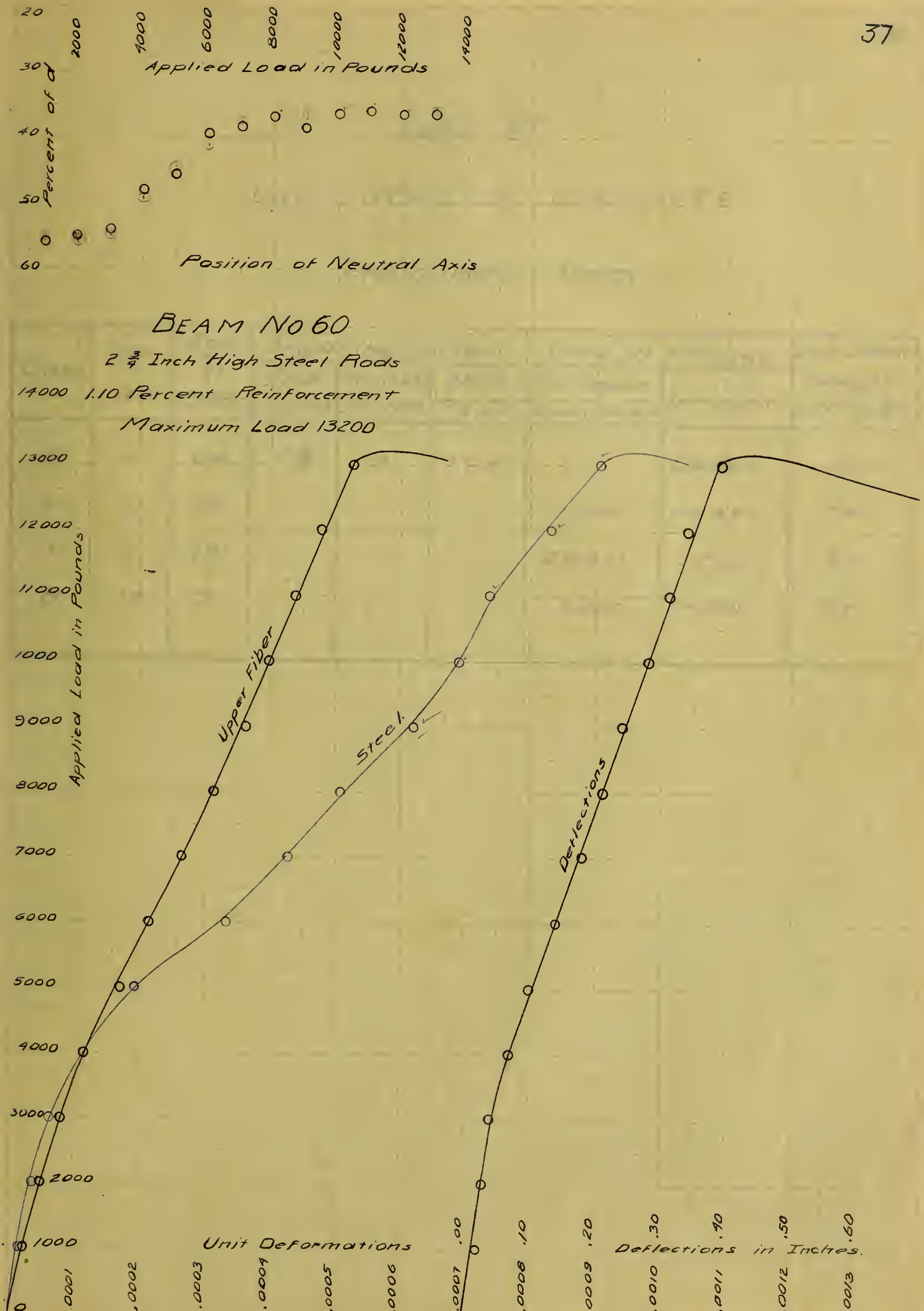


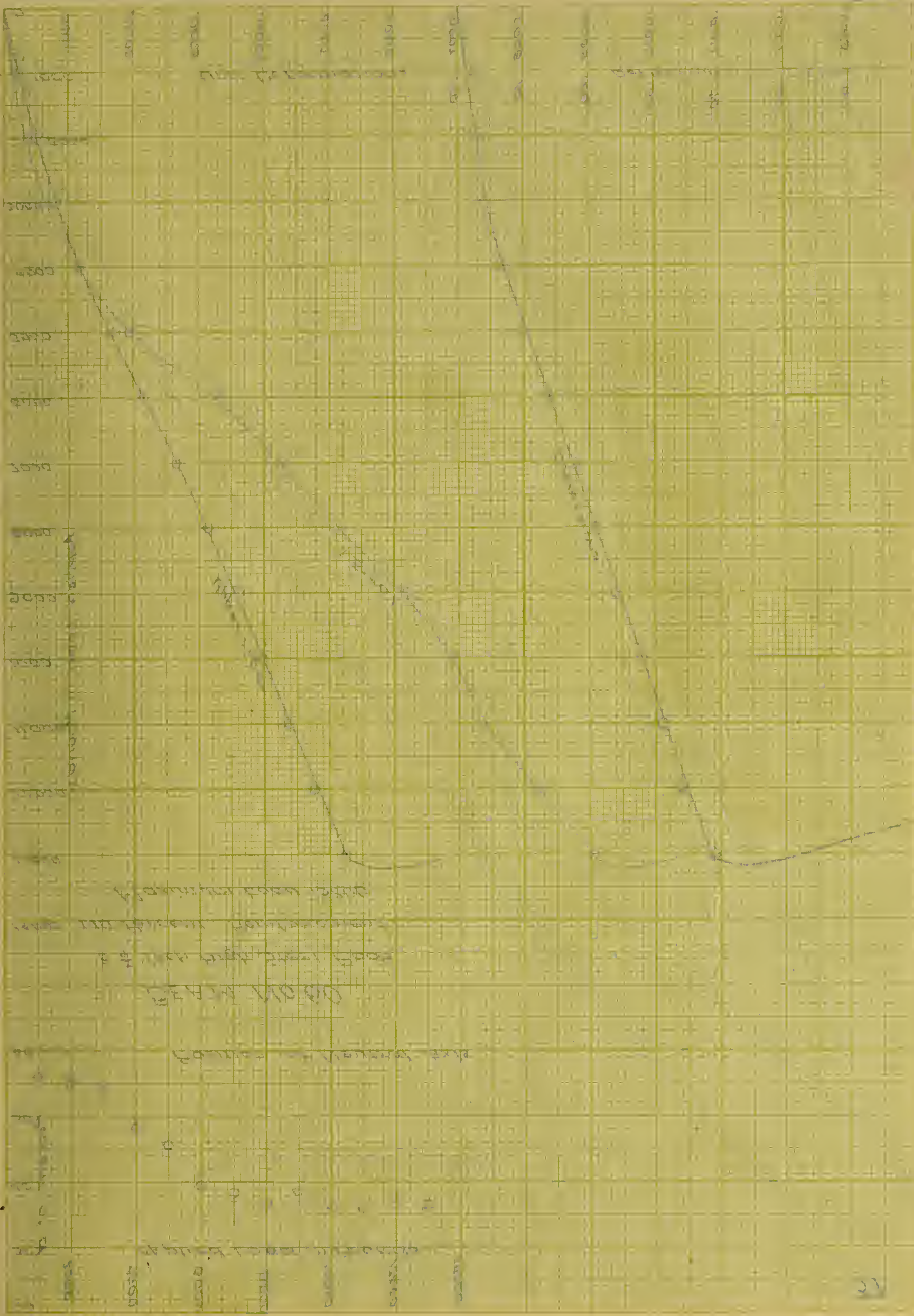


Applied Load in Pounds
Position of Neutral Axis
Percent of d
Broke
BEAM No 18
4 ½ Inch Reinforcement
Plain Round Rods
Percent Reinforcement = 0.98
Maximum Load 18800
Upper Fiber
Steel
Deflections
Applied Load in Pounds
Deflections in Inches
Unit Deformations

S+O.




Applied Load in Pounds
2000
4000
6000
8000
10000
12000
14000
Percent of d
20
30
40
50
60
Position of Neutral Axis
BEAM No 60
2 ¾ Inch High Steel Rods
1.10 Percent Reinforcement
Maximum Load 13200
Applied Load in Pounds
14000
13000
12000
11000
1000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
Upper Fiber
Steel
Deflections
Unit Deformations
.0001
.0002
.0003
.0004
.0005
.0006
.0007
.0008
.0009
.0010
.0011
.0012
.0013
Deflections in Inches.
.00
.10
.20
.30
.40
.50
.60
S.+O.



EUGENE DIETZGEN CO., CHICAGO.

Table VI

UNIT SHEAR OF CONCRETE

Preliminary Tests

Class	Ref. No	Age in Days	Shearing Section			Load in Pounds		Unit Shear Pounds per Sq. In.
			Dia. Inches	Thickness Inches	Area Sq. In.	First Appearance of Crack	Ultimate	
B	0	26	5⅞	3	55.4	—	29,000	523
A	5	19	"	"	"	10,000	24,400	440
A	7	19	"	"	"	25,000	35,000	632
B	38	7	"	"	"	8,000	24,800	447

Table VII

UNIT SHEAR OF CONCRETE SETTING IN WATER

Class "A"

Ref. No.	Age in Days	Shearing Section: Dia. Inches	Thickness Inches	Area Sq. In.	Load in Pounds: First Appearance of Crack	Ultimate	Unit Shear Pounds per Sq. In.
1	69	5 7/8	3-	55.4	25,000	39,500	713
2	69	"	2 15/16	54.2	24,000	33,000	609
3	69	"	2 15/16	54.2	25,000	36,000	664
4	69	"	3-	55.4	19,500	43,000	775
6	63	"	3-	55.4	25,000	37,100	670
8	61	"	3-	55.4	Cracked	40,000	722
9	61	"	2 15/16	54.2	20,000	52,500	968
10	61	"	3 1/16	56.5	21,000	47,200	886
11	61	"	3 1/16	56.5	24,000	50,000	885
12	61	"	3-	55.4	24,000	53,300	961
13	61	"	2 15/16	54.2	22,000	37,700	695
						Mean	777
Table VIII				**Class "B"**			
31	69	5 7/8	3 1/16	56.5	8,000	36,000	637
32	69	"	2 15/16	54.2	15,000	41,500	765
33	69	"	2 15/16	54.2	14,000	38400	708
34	61	"	2 15/16	54.2	12,000	36,200	667
35	61	"	3-	55.4	20,000	47,500	857
36	61	"	3 1/16	5.65	13,000	40,000	709
37	61	"	3-	55.4	17,000	42,000	758
						Mean -	729

Table IX

UNIT SHEAR OF CONCRETE SETTING IN AIR.

Class A

Ref No	Age in Days	Shearing Section			Load in Pounds		Unit Shear Pounds per Sq. In.
		Dia Inches	Thickness Inches	Area Sq. In	First Appearance of Crack	Ultimate	
14	60	5 7/8	3 1/16	56.5	23,000	51,000	903
15	60	"	3 1/16	56.5	23,000	36,700	650
16	60	"	3 -	55.4	30,000	52,500	947
17	60	"	2 15/16	54.2	21,000	34,800	642
18	61	"	3 1/16	56.5	0	39,000	691
19	61	"	3 -	55.4	24,000	49,500	894
20	61	"	2 15/16	54.2	22,000	52,500	968
21	61	"	3 -	55.4	26,000	49,000	894
22	60	"	3 1/16	56.5	19,000	47,000	832
23	60	"	3 1/16	56.5	0	37,000	655
24	60	"	3 1/16	56.5	23,000	51,500	911
25	60	"	3 1/16	56.5	19,000	41,000	725
26	60	"	2 15/16	54.2	14,000	32,500	600
27	61	"	3 1/16	56.5	17,500	45,500	805
28	61	"	3 1/16	56.5	18,000	40,500	717
29	61	"	3 1/16	56.5	24,000	50,000	885
30	61	"	3 -	55.4	22,000	45,500	811
						Mean-	796

Table X

UNIT SHEAR OF CONCRETE SETTING IN AIR

Class "B"

Ref No	Age in Days	Shearing Section			Load in Pounds -		Unit Shear Pounds per Sq. In.
		Dia. Inches	Thickness Inches	Area Sq. In.	First Appearance of Crack	Ultimate	
39	60	5 7/8	3 -	55.4	12,000	37,000	668
40	60	"	3 1/16	56.5	18,000	35,000	620
41	61	"	3 1/16	56.5	15,000	39,000	691
42	61	"	3 1/16	56.5	28,000	43,500	770
43	61	"	3 1/16	56.5	16,000	36,000	638
44	61	"	3 -	55.4	14,000	35,000	632
45	61	"	3 1/16	56.5	17,000	42,000	744
46	61	"	3 1/16	56.5	18,000	39,500	700
47	61	"	3 -	55.4	19,000	36,000	650
						Mean	679

Table XI Class "C."

Ref No	Age in Days	Dia. Inches	Thickness Inches	Area Sq. In.	First Appearance of Crack	Ultimate	Unit Shear Pounds per Sq. In.
48	59	5 7/8	3 1/16	56.5	28,000	47,000	831
49	59	"	3 1/16	56.5	36,000	65,800	1165
50	59	"	3 -	55.4	50,000	59,000	1065
51	59	"	3 1/16	56.4	52,000	64,500	1142
					Mean of	49,50,51.	1124

BEAM No 14

4 - ½ inch Plain Round Rods

Table XIII

Dimensions 11"×8"×13'-0" — Span 12'-0"

Applied Load in Pounds	Deflections in Inches	Extensometer Readings #1	#2	#3	#4	Deformations in Inches: Steel	Upper Fiber	Distance of Neutral Axis from Upp. Fib. in Inches
0	.00	.0165	.0072	.0077	.0112	.000000	.000000	4 —
1000	.01	.0191	.0110	.0104	.0145	(.000031 .000045	.000029) .000026	4.62
2000	.05	.0263	.0163	.0163	.0194	(.000067 .000087	.000057) .000117	4.55
3000	.11	.0353	.0224	.0241	.0255	(.000114 .000145	.000093) .000233	4.50
4000	.15	.0432	.0283	.0314	.0316	(.000167 .000221	.000148) .000345	4.72
5000	.20	.0505	.0345	.0386	.0379	(.000234 .000257	.000229) .000491	4.85
6000	.23	.0572	.0402	.0452	.0443	(.000290 .000319	.000322) .000500	5.20
7000	.28	.0634	.0455	.0516	.0506	(.000350 .000384	.000402) .000609	5.31
8000	.31	.0686	.0501	.0572	.0555	(.000402 .000428	.000472) .000682	5.40
9000	.35	.0757	.0562	.0638	.0622	(.000459 .000498	.000536) .000769	546
10000	.40	.0815	.0618	.0698	.0682	(.000534 .000557	.000636) .000893	547
11000	.43	.0875	.0678	.0756	.0745	(.000600 .000626	.000710) .000917	5.42
12000	.48	.0932	.0735	.0813	.0806	(.000662 .000693	.000780) .000991	5.40
13000	.51	.0995	.0795	.0875	.0866	(.000730 .000755	.000861) .001164	5.42
14000	.55	.1055	.0852	.0933	.0936	(.000790 .000822	.000933) .001192	5.40
15000	.61	.1113	.0910	.0989	.0981	(.000852 .000876	.001010) .001216	5.43
15400								

Note: For explanation of quantities in parenthesis, see under "Discussion of Results", page

BEAM No. 15

4 - ½ inch Plain Round Rods

Table XIV

Dimensions 11"×8"×13'-0" Span 12'-0"

Applied Load in Pounds	Deflections in Inches	Extensometer Readings				Deformations in Inches		Distance of Neutral Axis from Upp. Fib. in Inches
		#1	#2	#3	#4	Steel	Upper Fiber	
0	0	.0261	.0118	.0172	.0291	.000000	.000000	
1000	.02	.0284	.0142	.0195	.0319	.000029	.000026	4.5
2000	.03	.0308	.0172	.0219	.0350	.000062	.000057	4.45
3000	.05	.0334	.0202	.0248	.0381	.000105	.000081	4.40
4000	.06	.0365	.0241	.0282	.0422	.000152	.000121	4.45
5000	.08	.0402	.0288	.0324	.0465	.000200	.000174	4.55
6000	.11	.0442	.0340	.0368	.0510	.000257	.000221	4.55
7000	.14	.0484	.0394	.0412	.0559	.000322	.000266	4.53
8000	.17	.0528	.0453	.0457	.0614	.000393	.000314	4.44
9000	.21	.0574	.0514	.0505	.0669	.000464	.000372	4.44
10000	.25	.0620	.0573	.0549	.0722	.000533	.000424	4.42
11000	.28	.0665	.0633	.0594	.0775	.000600	.000471	4.40
12000	.31	.0713	.0698	.0649	.0823	.000665	.000531	4.41
13000	.34	.0767	.0758	.0696	.0885	.000738	.000590	4.43
14000	.38	.0815	.0822	.0749	.0941	.000805	.000650	4.45
15000	.41	.0865	.0882	.0800	.0995	.000878	.000712	4.48
16000	.45	.0917	.0949	.0854	.1055	.000952	.000771	4.50
17000	.48	.0968	.1016	.0906	.1115	.001025	.000833	4.49
17600	.52	.1012	.1068	.0948	.1159	.001077	.000881	4.50

BEAM No. 18

4 - $\frac{1}{2}$ inch Plain Round Rods

Dimensions 11" × 8" × 13'-0" — Table XV — Span 12'-0"

Applied Load in Pounds	Deflections in Inches	Extensometer Readings				Deformations in Inches		Distance of Neutral Axis from Upp. Fib. in Inches
		#1	#2	#3	#4	Steel	Upper Fiber	
0	.00	.0175	.0078	.0129	.0136	.000000	.000000	
1000	.03	.0201	.0105	.0149	.0161	.000029	.000026	4.50
2000	.06	.0234	.0146	.0175	.0191	.000071	.000060	4.40
3000	.08	.0273	.0195	.0213	.0234	.000129	.000107	4.48
4000	.10	.0312	.0247	.0255	.0286	.000190	.000150	4.39
5000	.13	.0358	.0305	.0308	.0347	.000262	.000207	4.37
6000	.17	.0401	.0363	.0357	.0406	.000331	.000260	4.36
7000	.20	.0443	.0420	.0409	.0467	.000407	.000310	4.34
8000	.25	?	.0478	.0463	.0527	.000476	.000378	4.40
9000	.28		.0540	.0520	.0593	.000550	.000448	4.47
10000	.31		.0600	.0576	.0655	.000615	.000519	4.55
11000	.36	New Zero	.0665	.0639	.0736	.000700	.000593	4.57
12000	.40	.0615	.0729	.0696	.0786	.000760	.000671	4.67
13000	.45	.0671	.0799	.0761	.0859	.000848	.000736	4.66
14000	.49	.0720	.0871	.0826	.0927	.000925	.000810	4.67
15000	.54	.0792	.0943	.0892	.0997	.001058	.000840	4.68
16000	.58	.0862	.1025	.0969	.1076	.001158	.000976	4.68
17000	.62	.0935	.1063	.1037	.1151	.001258	.001063	4.69
18000	.67	.1027	.1229	.1121	.1249	.001381	.001162	4.68
18800								

Note: Extensometer Bar #1 slipped off roller at 8000#

BEAM No 60

2-$\frac{3}{4}$" inch Plain Round Rods.

Dimensions 11"×8"×13'0"   Table XVI   Span 12' 0"

Applied Load in Pounds	Deflections in Inches	Extensometer Readings				Deformations in Inches.		Distance of Neutral Axis from Upper Fiber in Inches
		#1	#2	#3	#4	Steel	Upper Fiber	
0	0	.0059	.0055	.0071	.0042	.0000	.0000	
1,000	.02	.0074	.0070	.0090	.0060	.000017	.000074	5.60
2,000	.03	.0095	.0091	.0113	.0081	.000038	.000050	5.50
3,000	.04	.0119	.0119	.0148	.0111	.000063	.000083	5.40
4,000	.07	.0155	.0161	.0172	.0152	.000119	.000119	4.80
5,000	.10	.0199	.0220	.0218	.0211	.000195	.000172	4.55
6,000	.14	.0260	.0320	.0283	.0323	.000338	.000217	3.95
7,000	.18	.0304	.0390	.0329	.0392	.000429	.000264	3.85
8,000	.21	.0352	.0465	.0379	.0470	.000507	.000314	3.70
9,000	.24	.0392	.0537	.0432	.0532	.000620	.000362	3.88
10,000	.28	.0432	.0585	.0464	.0592	.000690	.000398	3.65
11,000	.31	.0473	.0642	.0504	.0650	.000738	.000438	3.60
12,000	.34	.0512	.07000	.0544	.0708	.000831	.000476	3.64
13,000	.39	.0553	.0755	.0588	.0763	.000905	.000524	3.65
13,200								